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NOTES ON THE TRANSIENT RESPONSE AND ERRORS OF SHOCK ACCELEROMET--ETC(U)
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NOTES ON THE TRANSIENT RESPONSE AND ERRORS OF SHOCK ACCELEROMETERS WITH SECOND-ORDER FILTERS

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W. R. Macdonald

SUMMARY

The response of accelerometers, with second-order filters, is considered for ½ sin and $\sin^2$ acceleration pulses. The time history of their integrated (velocity) response is also treated.

Graphs relating the error in measuring peak acceleration to the filter parameters are given, and the results are related to practical situations experienced when calibrating accelerometers in RAE.

The effect of accelerometer non-linearity when calibrating by velocity-change methods is briefly investigated and a method of measuring the non-linearity is suggested.

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1 INTRODUCTION

These notes are intended to serve as an aid to quantifying some of the errors which arise in the calibration and use of shock accelerometers. Calibration errors associated with an RAE airgun facility[1] have already been considered[2] particularly in the context of signal-conditioning and calibration equipment. Nevertheless, Ref 2 identified important sources of error related to the characteristics of the accelerometer under test and it is felt that some of these warrant further consideration.

Accelerometers for shock measurement must, in general, have sufficient acceleration range and frequency response to register the shock waveform adequately. Thus, most accelerometers used for measuring shocks of up to say $10^5 g_n$, are of the piezoelectric type with undamped natural frequencies of the order of 100 kHz. These, being undamped , are likely to ring when subjected to shock and moreover, other resonances associated with the mounting of the instrument are likely to colour the output waveform. Such resonances, more often than not, completely obscure the shock waveform and hence it is usual to place a filter in the measurement system in order to obtain what is hoped to be an adequately faithful representation of the shock pulse. The choice of filter and where to put it, is more art than science.

Filters may be electrical or mechanical. The former are usually employed since they are easy to use and are readily adjustable to suit the prevailing conditions. But extreme caution must be exercised lest they mask overloading of the accelerometer, or its conditioning unit, by inputs whose spectrum contains frequency components outside the pass band of the filter. It is therefore better, in principle, to introduce a mechanical filter at the very beginning of the measurement system. Such filters are available commercially in the form of accelerometer mounts, but their use is limited to small-shock measurements. Simple calculations quickly reveal the problems of designing such filters with useful amounts of damping to withstand shocks of $10^5 g_n$. Nevertheless, it is the writer's belief that such filters must be developed and used if shock signatures are to be more reliably measured.

Because the characteristics of mechanical filters at high shock levels are unlikely to lend themselves to accurate control, there is a need to quantify the performance of accelerometers with such filters. For reasons of simplicity, the treatment below is confined to second-order filters and to shock pulses of $\frac{1}{2}$ sin and $\sin^2$ form. Many shock pulses, and particularly those generated by the airgun used for shock calibration, are roughly of these forms.

A special case of a mechanical filter exists when calibrating by the comparison method[2] in which the test accelerometer is mounted on a reference transducer. Errors can arise from so-called 'relative motion' between the sensing element of the reference transducer and the surface on which the test accelerometer is mounted. The coupling between the reference and test accelerometer can be represented with fair accuracy by a second-order mechanical filter whose undamped natural frequency and damping ratio depend on the design of the reference transducer and the mass of the test accelerometer. This case is analysed in the present paper.

For completeness, the response of filtered accelerometers with integrators is also considered. This relates to calibration by the velocity method[2] in which the integrated output of the accelerometer is equated to the measured velocity change. The use of such data need not, in fact, be confined to calibration but may be extended to practical measurement situations where the velocity change is known or can be guessed with fair accuracy. In such cases, the extent to which the integrated output of the accelerometer agrees with the known velocity change can provide a valuable indication of the integrity of measurement.

Whilst emphasis is given to presenting the output-time relationships of accelerometers and deriving the associated errors, the effects of accelerometer non-linearity are briefly considered in the context of velocity calibration. Piezoelectric accelerometers tend to become more sensitive at high levels of acceleration and establishing non-linearity is part of the calibration procedure.

2 RESPONSE TO $\frac{1}{2}$ SIN PULSE

Consider a $\frac{1}{2}$ sin pulse (Fig 1a) where the instantaneous acceleration is given by

$$a(t) = \hat{a} \sin \omega t \qquad 0 < t \leqslant \pi/\omega \qquad (1)$$

$$= 0 \qquad t > \pi/\omega \qquad (1a)$$

where $\hat{a}$ = peak acceleration

$\omega = \pi/T$

T = period of pulse

and t = time .

If this pulse is passed through a second-order filter of transfer function

$$h(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \tag{2}$$

where $h(s)$ = transfer function of filter

$\omega_n = 2\pi f_n$

f_n = undamped natural frequency

and ζ = damping ratio (assumed < 1)

it is shown in Appendix A that the output of the accelerometer and filter is

$$y(t) = g(t) + g\left(t - \frac{\pi}{\omega}\right)H\left(t - \frac{\pi}{\omega}\right) \tag{3}$$

where

$$g(t) = k\hat{a}\left[A\cos\omega t + \frac{B}{\omega}\sin\omega t + Ce^{-\zeta\omega_n t}\cos\omega_n(1-\zeta^2)^{\frac{1}{2}}t + \frac{D - C\zeta\omega_n}{\omega_n(1-\zeta^2)^{\frac{1}{2}}}e^{-\zeta\omega_n t}\sin\omega_n(1-\zeta^2)^{\frac{1}{2}}t\right] \tag{3a}$$

$$H\left(t - \frac{\pi}{\omega}\right) = 0 \quad \text{for } t < \frac{\pi}{\omega}$$

$$= 1 \quad \text{for } t > \frac{\pi}{\omega}$$

k = sensitivity of accelerometer

$$A = \frac{-2\zeta\omega\omega_n^3}{\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2} \tag{3b}$$

$$B = \frac{\omega\omega_n^2\left(\omega_n^2 - \omega^2\right)}{\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2} \tag{3c}$$

$$C = -A \tag{3d}$$

and

$$D = \frac{\omega_n^4\left(\omega^2 - \omega_n^2\right)}{\omega\left[\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2\right]} + \frac{\omega_n^2}{\omega} \ . \tag{3e}$$

Equation (3) gives the acceleration-time history of the filtered accelerometer output.

If a unity-gain integrator, of transfer function $\frac{1}{s}$, follows the accelerometer and filter, it is shown in Appendix A that its output, which defines the velocity-time history, is given by

$$v(t) = g(t) + g\left(t - \frac{\pi}{\omega}\right)H\left(t - \frac{\pi}{\omega}\right) \tag{4}$$

and in this case

$$g(t) = k\hat{a}\Bigg[A + B\cos\omega t + \frac{C}{\omega}\sin\omega t + De^{-\zeta\omega_n t}\cos\omega_n(1-\zeta^2)^{\frac{1}{2}}t + \frac{E - D\zeta\omega_n}{\omega_n(1-\zeta^2)^{\frac{1}{2}}}e^{-\zeta\omega_n t}\sin\omega_n(1-\zeta^2)^{\frac{1}{2}}t\Bigg] \tag{4a}$$

with

$$A = \frac{1}{\omega} \tag{4b}$$

$$B = \frac{\left(\omega^2 - \omega_n^2\right)\omega_n^2}{\omega\left[\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2\right]} \tag{4c}$$

$$C = -\frac{2\zeta\omega\omega_n^3}{\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2} \tag{4d}$$

$$D = -B - A$$

and

$$E = \frac{2\zeta\omega_n^5}{\omega\left[\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2\right]} - \frac{2\zeta\omega_n}{\omega} \ . \tag{4e}$$

If only the final value of the velocity output is required, this may be obtained from equation (A-3) by using the final-value theorem of Laplace transforms which states that

$$\lim_{t\to\infty} v(t) = \lim_{s\to\infty} sv(s)$$

whence

$$\lim_{t\to\infty} v(t) = 2\frac{kT}{\pi}\hat{a} \quad . \tag{5}$$

Thus the final value of the integrated output is independent of the characteristics of the (low-pass) filter and no account need be taken of it when calibrating by the velocity method other than to ensure that the signal has settled to its final value before shutting off the integration.

By way of example, Fig 2 shows the response of an accelerometer with filter of $f_n = 28$ kHz and $\zeta = 0.01$ to a $\frac{1}{2}$ sin pulse of unit acceleration and of length 100 μs . The sensitivity of the accelerometer is assumed to be unity, and for these conditions it is seen that if the peak output were recorded on a peak-reading meter the value recorded would have an error of about 14%. Moreover, a considerable period would have to elapse before the velocity output attained a steady value.

Fig 3 shows the error in measuring peak acceleration for various values of ω_n (in terms of T) and ζ , whilst Fig 4a gives the approximate times for the integrated output to settle within 1% of its final value. It is evident from Fig 3 that the error is very small for values of $(\omega_n T)/\pi > 4$ if $\zeta = 1/\sqrt{2}$ (Butterworth filter).

3 RESPONSE TO SIN^2 PULSE

Whereas the $\frac{1}{2}$ sin pulse is a useful shock waveform for calibration purposes, under severe impacts the sharp corners at the beginning and end of the waveform can cause excessive excitation of the transducer resonance, or other resonances associated with the transducer mounting. Thus, in calibration, it is usual to place a wad of felt or other crushable material on one of the impacting surfaces. This gives a shock waveform not unlike a $\sin^2$ pulse (Fig 1b).

For a $\sin^2$ pulse the acceleration is

$$a(t) = \hat{a}\sin^2\omega t \qquad 0 < t \leqslant \pi/\omega \tag{6}$$

$$= 0 \qquad t > \pi/\omega \quad . \tag{6a}$$

From Appendix B, the output of the accelerometer and filter is

$$y(t) = g(t) - g\left(t - \frac{\pi}{\omega}\right)H\left(t - \frac{\pi}{\omega}\right) \tag{7}$$

with

$$g(t) = 2k\hat{a}\left[A + B\cos 2\omega t + \frac{C}{2\omega}\sin 2\omega t \right.$$

$$\left. + De^{-\zeta\omega_n t}\cos\omega_n(1-\zeta^2)^{\frac{1}{2}}t + \frac{E - D\zeta\omega_n}{\omega_n(1-\zeta^2)^{\frac{1}{2}}}e^{-\zeta\omega_n t}\sin\omega_n(1-\zeta^2)^{\frac{1}{2}}t\right] \tag{7a}$$

and

$$A = \frac{1}{4} \tag{7b}$$

$$B = \frac{\omega_n^2\left(4\omega^2 - \omega_n^2\right)}{4\left[\left(4\omega^2 - \omega_n^2\right)^2 + 16\zeta^2\omega^2\omega_n^2\right]} \tag{7c}$$

$$C = -\frac{2\zeta\omega^2\omega_n^3}{\left(4\omega^2 - \omega_n^2\right)^2 + 16\zeta^2\omega^2\omega_n^2} \tag{7d}$$

$$D = -A - B \tag{7e}$$

$$E = \frac{\zeta\omega_n}{2}\left[\frac{\omega_n^4}{\left(4\omega^2 - \omega_n^2\right)^2 + 16\zeta^2\omega^2\omega_n^2} - 1\right] . \tag{7f}$$

If the accelerometer and filter are followed by an integrator, the velocity output is

$$v(t) = g(t) - g\left(t - \frac{\pi}{\omega}\right)H\left(t - \frac{\pi}{\omega}\right) \tag{8}$$

with

$$g(t) = 2k\hat{a}\left[At + B + C\cos 2\omega t + \frac{D}{2\omega}\sin 2\omega t \right.$$

$$\left. + Ee^{-\zeta\omega_n t}\cos\omega_n(1-\zeta^2)^{\frac{1}{2}}t + \frac{F - E\zeta\omega_n}{\omega_n(1-\zeta^2)^{\frac{1}{2}}}e^{-\zeta\omega_n t}\sin\omega_n(1-\zeta^2)^{\frac{1}{2}}t\right] \tag{8a}$$

and

$$A = \frac{1}{4} \tag{8b}$$

$$B = -\frac{\zeta}{2\omega_n}$$

$$C = \frac{\zeta\omega_n^3}{2\left[\left(4\omega^2 - \omega_n^2\right)^2 + 16\zeta^2\omega^2\omega_n^2\right]} \tag{8c}$$

$$D = \frac{\omega_n^2\left(4\omega^2 - \omega_n^2\right)}{4\left[\left(4\omega^2 - \omega_n^2\right)^2 + 16\zeta^2\omega^2\omega_n^2\right]} \tag{8d}$$

$$E = \frac{4\zeta\omega^2\left(2\omega^2 - \omega_n^2 + 2\zeta^2\omega_n^2\right)}{\omega_n\left[\left(4\omega^2 - \omega_n^2\right)^2 + 16\zeta^2\omega^2\omega_n^2\right]} \tag{8e}$$

$$F = \frac{\omega^2\left(16\omega^2\zeta^2 - 12\zeta^2\omega_n^2 + 16\zeta^4\omega_n^2 - 4\omega^2 + \omega_n^2\right)}{\left(4\omega^2 - \omega_n^2\right)^2 + 16\zeta^2\omega^2\omega_n^2} \quad . \tag{8f}$$

The final value of the velocity output is again independent of the filter parameters and is given by

$$\lim_{t\to\infty} v(t) = \frac{kT}{2}\hat{a} \ . \tag{9}$$

Time histories of the acceleration and velocity outputs for $T = 100\ \mu s$, $f_n = 28$ kHz and $\zeta = 0.01$ are shown in Fig 5. Comparison with the response to a $\frac{1}{2}$ sin pulse (Fig 2) shows that both the ringing and peak error are considerably smaller.

The error in measuring peak acceleration, and the time taken for the integrated output to settle to within 1% of its final value, are shown in Figs 6 and 4b respectively. The error is small for values of $(\omega_n T)/\pi > 4$ if $\zeta = 0.67$. This value of damping ratio is close to that of the so-called 'optimum damping' value used in transducers.

4 CALCULATION OF ACCELEROMETER NON-LINEARITY WHEN CALIBRATING BY THE VELOCITY METHOD

Piezoelectric sensors in general display a high degree of linearity and so have a very wide dynamic range. Nevertheless, when called upon to measure very large accelerations non-linearity may become significant in transducers with ceramic elements.

In practice, the sensitivity tends to increase linearly with acceleration input so that it has the form

$$k' = k\left(1 + \lambda a(t)\right) \tag{10}$$

where k' = true sensitivity

k = low-level sensitivity

and λ = fractional change of sensitivity per unit input acceleration.

Fig 7 shows a sketch of the sensitivity and input-output relationship with the non-linearity enhanced for clarity. Clearly, non-linearity can cause problems when calibrating by the velocity method since point-by-point calibration cannot be performed. However, if λ is constant (often a fair assumption) then its value can be calculated if the shape and duration of the pulse is known.

4.1 Calculation of λ when calibrating with $\frac{1}{2}$ sin pulse

For a $\frac{1}{2}$ sin pulse, it has already been shown (equation (5)) that the final value of the integrated output is, with integrator of transfer function $\frac{1}{s}$ $\left(\text{gain} = \left|\frac{1}{\omega}\right|\right)$,

$$\underset{t\to\infty}{v(t)} = 2\,\frac{kT}{\pi}\,\hat{a}\ . \tag{11}$$

If, however, the gain is k', the final output will be, from equations (1) and (10),

$$\underset{t\to\infty}{v(t)} = \hat{a}\int_0^{\pi/\omega} k'\sin\omega t\,dt$$

$$= k\hat{a}\int_0^{\pi/\omega} \left(\sin\omega t + \lambda\hat{a}\sin^2\omega t\right)dt$$

$$= kT\hat{a}\left(\frac{2}{\pi} + \frac{\lambda\hat{a}}{2}\right) . \quad (12)$$

The fractional error, caused by non-linearity, in the final value of the integrated output is therefore, from equations (11) and (12),

$$E = \frac{\pi\lambda\hat{a}}{4} \quad (13)$$

where E = fractional error in velocity measurement caused by non-linearity.

To gain an estimate of the value of λ from a velocity calibration, the output recorded at one or more high acceleration levels must be related to the low-level sensitivity k obtained, for example, by calibrating on a vibrator. Equating the final value of the integrated output with the measured velocity change gives, from equations (11) and (12)

$$\underset{t\to\infty}{v(t)} = kT\hat{a}\left(\frac{2}{\pi} + \frac{\lambda\hat{a}}{2}\right)$$

with

$$\hat{a} = \frac{\Delta v\pi}{2T}$$

where Δv = measured velocity change.

Hence

$$\lambda = \frac{8T\left(\underset{t\to\infty}{v(t)} - k\Delta v\right)}{\pi^2 k(\Delta v)^2} . \quad (14)$$

4.2 <u>Calculation of λ when calibrating with $\sin^2$ pulse</u>

For a $\sin^2$ pulse, the final value of the integrated output is, from equation (9),

$$\underset{t\to\infty}{v(t)} = \frac{kT\hat{a}}{2} . \quad (15)$$

If the gain is k', the final output is, from equations (6) and (10),

$$\underset{t\to\infty}{v(t)} = k\hat{a} \int_0^{\pi/\omega} (\sin^2\omega t + \lambda\hat{a}\sin^4\omega t)dt$$

$$= \frac{kT\hat{a}}{2}\left(1 + \frac{3\lambda\hat{a}}{4}\right) \quad . \tag{16}$$

The fractional error is therefore, from equations (15) and (16), given by

$$E = \frac{3\lambda\hat{a}}{4} \quad . \tag{17}$$

From equations (15) and (16),

$$\underset{t\to\infty}{v(t)} = \frac{kT\hat{a}}{2}\left(1 + \frac{3\lambda\hat{a}}{4}\right)$$

with

$$\hat{a} = \frac{2\Delta v}{T} \quad .$$

Hence

$$\lambda = \frac{2T\left(\underset{t\to\infty}{v(t)} - k\Delta v\right)}{3k(\Delta v)^2} \quad . \tag{18}$$

Calculation of λ in either case has involved a knowledge of the pulse shape and period. Whilst the latter may be measured with fair accuracy, the former must be a matter of qualitative assessment. Comparing equations (14) and (18) shows that the values of λ for ½ sin and $\sin^2$ pulses have coefficients of $8/(\pi^2)$ and 2/3 respectively. Hence the value of λ could be in error by as much as 20% were the pulse shape wrongly assessed. However, since λ is, or should be, small, errors in its measurement should not be of great significance. Typically, the value of λ is such that k' at the upper limit of range is 5% higher than k .

The above treatment assumes that λ is constant and independent of acceleration. The validity of this assumption can, of course, be confirmed by performing velocity calibrations at a number of acceleration levels.

5 EXAMPLES RELATING TO THE AIRGUN USED FOR SHOCK ACCELEROMETER CALIBRATION

To illustrate some applications of the data derived in previous sections, typical examples relating to the calibration of accelerometers in RAE with an airgun facility are considered here. The airgun produces an acceleration pulse of up to 10^6 ms^{-2} with a period of about 100 μs.

5.1 Relative-motion error when calibrating by the comparison method

In calibrating by the comparison method, the test accelerometer is mounted on the end face of a special reference accelerometer which has been previously calibrated. The peak outputs of the test and reference accelerometers are recorded on peak-reading voltmeters and the readings compared. This method is both simple and reliable provided that the test accelerometer experiences the same input acceleration as the sensor in the reference. Errors may arise, however, because there is in practice some relative motion between the test accelerometer and sensor.

It has been deduced from the manufacturer's data, and confirmed by frequency-response measurements on a vibrator, that the mechanical couplings between the accelerometer and reference transducer normally used may be represented by a second-order system having a natural frequency of 29 kHz and damping ratio of 0.49. Using these values in equations (3)-(3e) and (7)-(7f) shows that the peak acceleration applied to the test accelerometer is 1.4% higher than that experienced by the reference for a $\frac{1}{2}$ sin pulse and 3.0% higher for a $\sin^2$ pulse. Thus, although a $\sin^2$ pulse produces less ringing than a $\frac{1}{2}$ sin pulse, the relative-motion error may be greater. Nevertheless, relative-motion errors are, in general, small.

5.2 Error caused by accelerometer resonance

Shock accelerometers typically have an undamped natural frequency of about 100 kHz with small damping. Thus, ringing will occur and the amplitude of the peak registered will be in error. Fig 8 shows the error in measuring the peak acceleration of $\frac{1}{2}$ sin and $\sin^2$ pulses of 100 μs duration for transducer natural frequencies ranging from 50-150 kHz, assuming a damping ratio of 0.01. Since the damping ratio is very small, the greater ringing excited by the $\frac{1}{2}$ sin pulse produces larger errors than those with a $\sin^2$ pulse. The effects of accelerometer resonance can often be reduced by careful use of electrical filters.

6 CONCLUSIONS

It is hoped that the data presented in this Memorandum may afford the reader a better picture of some aspects of the performance of shock accelerometers and provide a means of quantifying some of the errors which can arise during calibration. Moreover, the data on filtered response should be useful in applications where mechanical filters are deliberately placed in the measurement chain.

More specifically, it has been shown that:-

(1) The best filter for a $\frac{1}{2}$ sin pulse is one with a damping ratio of $1/\sqrt{2}$, whilst for $\sin^2$ pulses a damping ratio of about 0.67 is the optimum.

(2) The parameters of the filter do not affect the final value of the integral of the accelerometer output.

(3) The error associated with relative motion when calibrating by the comparison method is unlikely to exceed 3% with the accelerometer and reference transducer presently used in RAE.

(4) It is fairly easy, in principle, to measure an approximate value of accelerometer non-linearity when calibrating by the velocity method, provided that the accelerometer sensitivity increases or decreases linearly with acceleration.

Clearly, caution is necessary in dealing with practical situations where mechanical filters may display characteristics other than those of a second-order system, and where the shock differs significantly from the two ideal pulse shapes considered.

Appendix A

RESPONSE TO ½ SIN PULSE

For an input of $a(t) = \hat{a} \sin \omega t \,, \qquad 0 < t \leqslant \frac{\pi}{\omega}$

$$= 0 \,, \qquad t > \frac{\pi}{\omega}$$

$$= \hat{a}\left\{\sin \omega t + \left[\sin \omega\left(t - \frac{\pi}{\omega}\right)\right]\left[H\left(t - \frac{\pi}{\omega}\right)\right]\right\}$$

its Laplace transform is

$$a(s) = \hat{a} \frac{\omega}{s^2 + \omega^2}\left[1 + e^{-(s\pi)/\omega}\right] .$$

If a second-order filter of transfer function

$$h(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

is placed before, or after, the accelerometer of sensitivity k , the transform of the output is

$$y(s) = k\hat{a} \frac{\omega\omega_n^2}{\left(s^2 + \omega^2\right)\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}\left[1 + e^{-(s\pi)/\omega}\right] \qquad \text{(A-1)}$$

$$= k\hat{a}\left[\frac{As + B}{s^2 + \omega^2} + \frac{Cs + D}{s^2 + 2\zeta\omega_n s + \omega_n^2}\right]\left[1 + e^{-(s\pi)/\omega}\right]$$

$$= k\hat{a}\left[\frac{As + B}{s^2 + \omega^2} + \frac{C(s + \zeta\omega_n) + D - C\zeta\omega_n}{\left(s + \zeta\omega_n\right)^2 + \omega_n^2(1 - \zeta^2)}\right]\left[1 + e^{-(s\pi)/\omega}\right]$$

with the coefficients of the partial fractions being

$$A = -\frac{2\zeta\omega\omega_n^3}{(\omega^2 - \omega_n^2)^2 + 4\zeta^2\omega^2\omega_n^2}$$

$$B = \frac{\omega\omega_n^2(\omega_n^2 - \omega^2)}{(\omega^2 - \omega_n^2)^2 + 4\zeta^2\omega^2\omega_n^2}$$

$$C = -A$$

and

$$D = \frac{\omega_n^4(\omega^2 - \omega_n^2)}{\omega\left[(\omega^2 - \omega_n^2)^2 + 4\zeta^2\omega^2\omega_n^2\right]} \,.$$

Inverting the transform gives

$$a(t) = k\hat{a}\left[A\cos\omega t + \frac{B}{\omega}\sin\omega t + Ce^{-\zeta\omega_n t}\cos\omega_n(1-\zeta^2)^{\frac{1}{2}}t + \frac{D - C\zeta\omega_n}{\omega_n(1-\zeta^2)^{\frac{1}{2}}}e^{-\zeta\omega_n t}\sin\omega_n(1-\zeta^2)^{\frac{1}{2}}t\right] + \left[g\left(t-\frac{\pi}{\omega}\right)H\left(t-\frac{\pi}{\omega}\right)\right] \quad \text{(A-2)}$$

with $g\left(t - \frac{\pi}{\omega}\right)$ being the first term in the square brackets with $\left(t - \frac{\pi}{\omega}\right)$ substituted for t, and $H\left(t - \frac{\pi}{\omega}\right)$ being Heaviside's shifting function.

If an integrator is added to the system, equation (A-1) becomes

$$y(s) = k\hat{a}\frac{\omega\omega_n^2}{s(s^2+\omega^2)(s^2+2\zeta\omega_n s+\omega_n^2)}\left[1 + e^{-(s\pi)/\omega}\right]. \quad \text{(A-3)}$$

This may be rewritten as

$$y(s) - k\hat{a}\left[\frac{A}{s} + \frac{Bs + C}{s^2+\omega^2} + \frac{D(s+\zeta\omega_n) + E - D\zeta\omega_n}{(s+\zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}\right]\left[1 + e^{-(s\pi)/\omega}\right]$$

with the coefficients being

$$A = \frac{1}{\omega}$$

$$B = \frac{\left(\omega^2 - \omega_n^2\right)\omega_n^2}{\omega\left[\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2\right]}$$

$$C = -\frac{2\zeta\omega\omega_n^3}{\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2}$$

$$D = -B - A$$

and

$$E = \frac{2\zeta\omega_n^5}{\omega\left[\left(\omega^2 - \omega_n^2\right)^2 + 4\zeta^2\omega^2\omega_n^2\right]} .$$

Inverting the transform gives the velocity output of

$$v(t) = k\hat{a}\left[A + B\cos\omega t + \frac{C}{\omega}\sin\omega t + De^{-\zeta\omega_n t}\cos\omega_n(1 - \zeta^2)^{\frac{1}{2}}t + \frac{E - D\zeta\omega_n}{\omega_n(1-\zeta^2)^{\frac{1}{2}}}e^{-\zeta\omega_n t}\sin\omega_n(1 - \zeta^2)^{\frac{1}{2}}t\right] + \left[g\left(t - \frac{\pi}{\omega}\right)H\left(t - \frac{\pi}{\omega}\right)\right] .$$

Appendix B

RESPONSE TO SIN^2 PULSE

For a $\sin^2$ pulse, the input is

$$
\begin{aligned}
a(t) &= \hat{a}\sin^2\omega t\ , \qquad 0 < t \leqslant \frac{\pi}{\omega} \\
&= 0 \qquad\qquad\ \ , \qquad t > \frac{\pi}{\omega}\ .
\end{aligned}
$$

Since

$$\hat{a}\sin^2\omega t = \frac{\hat{a}}{2}\left(1 - \cos^2\omega t\right)\ ,$$

$$a(t) = \frac{\hat{a}}{2}\left\{\left[1 - \cos 2\omega t\right] - \left[1 - \cos 2\omega\left(t - \frac{\pi}{\omega}\right)\right]\left[H\left(t - \frac{\pi}{\omega}\right)\right]\right\}\ .$$

Hence

$$
\begin{aligned}
a(s) &= \frac{\hat{a}}{2}\left\{\left[\frac{1}{s} - \frac{s}{s^2 + 4\omega^2}\right]\left[1 - e^{-(s\pi)/\omega}\right]\right. \\
&= \frac{\hat{a}}{2}\,\frac{4\omega^2}{s(s^2 + 4\omega^2)}\left[1 - e^{-(s\pi)/\omega}\right]\ .
\end{aligned}
$$

With the addition of a second-order filter the transform of the output becomes

$$y(s) = 2\hat{a}\,\frac{\omega^2\omega_n^2}{s(s^2 + 4\omega^2)\left(s^2 + 2\zeta\omega_n s + \omega_n^2\right)}\left[1 - e^{-(s\pi)/\omega}\right]\ .$$

Splitting this into partial fractions, as performed in Appendix A, and inverting the transform leads to the output-time relationship described by equation (7). A similar process leads to equation (8) describing the velocity relationship.

REFERENCES

No.	Author	Title, etc
1	J. Knight	An airgun for the calibration of shock accelerometers. RAE Technical Report (to be published)
2	J.W. Stemp	The calibration of shock accelerometers up to 100000 g_n RAE Technical Report (to be published)

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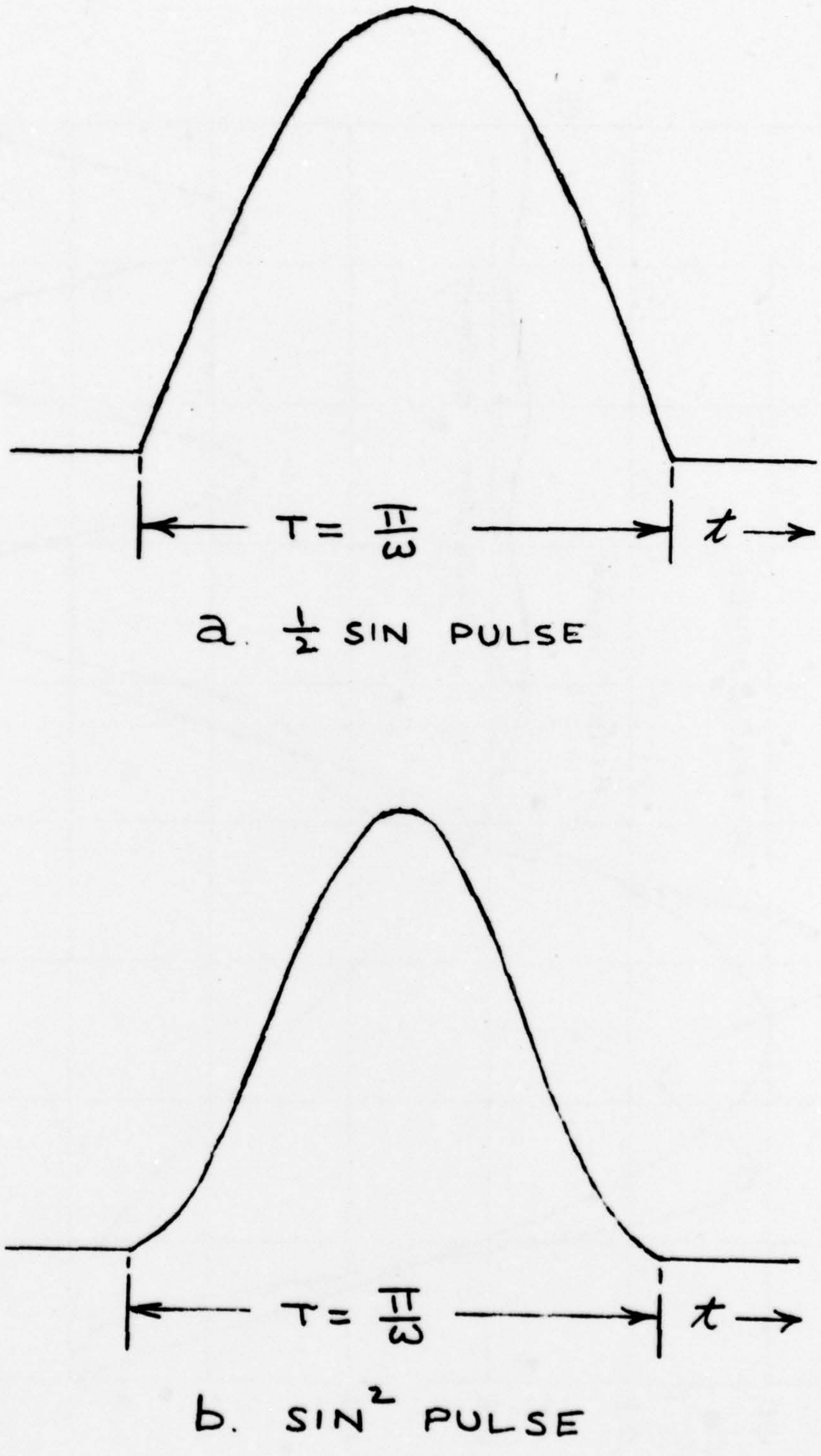


Fig 1 Nature of shock pulses

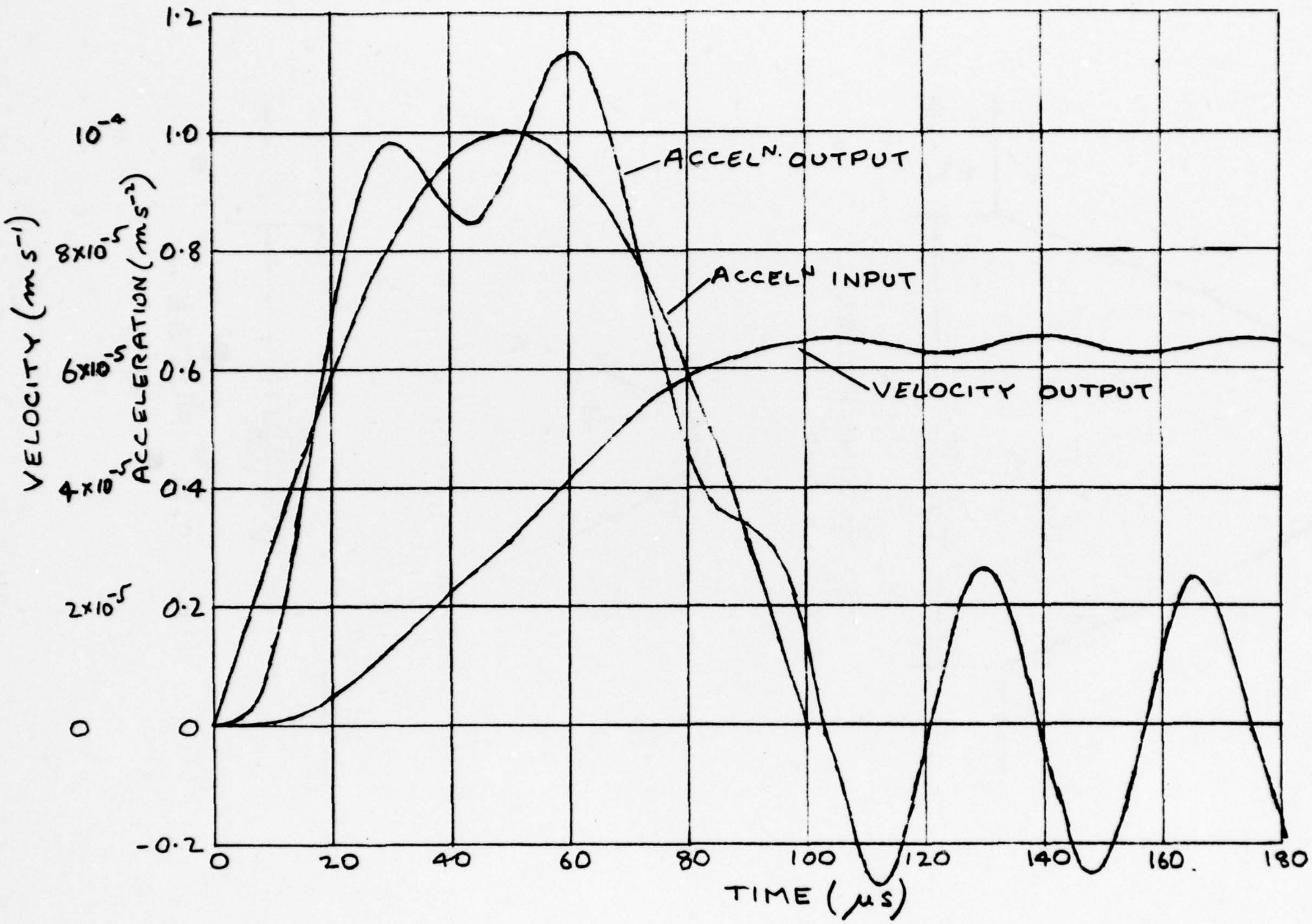


Fig 2 Response to ½ sin pulse (T = 100 μs, f_n = 28 kHz and ζ = 0.01)

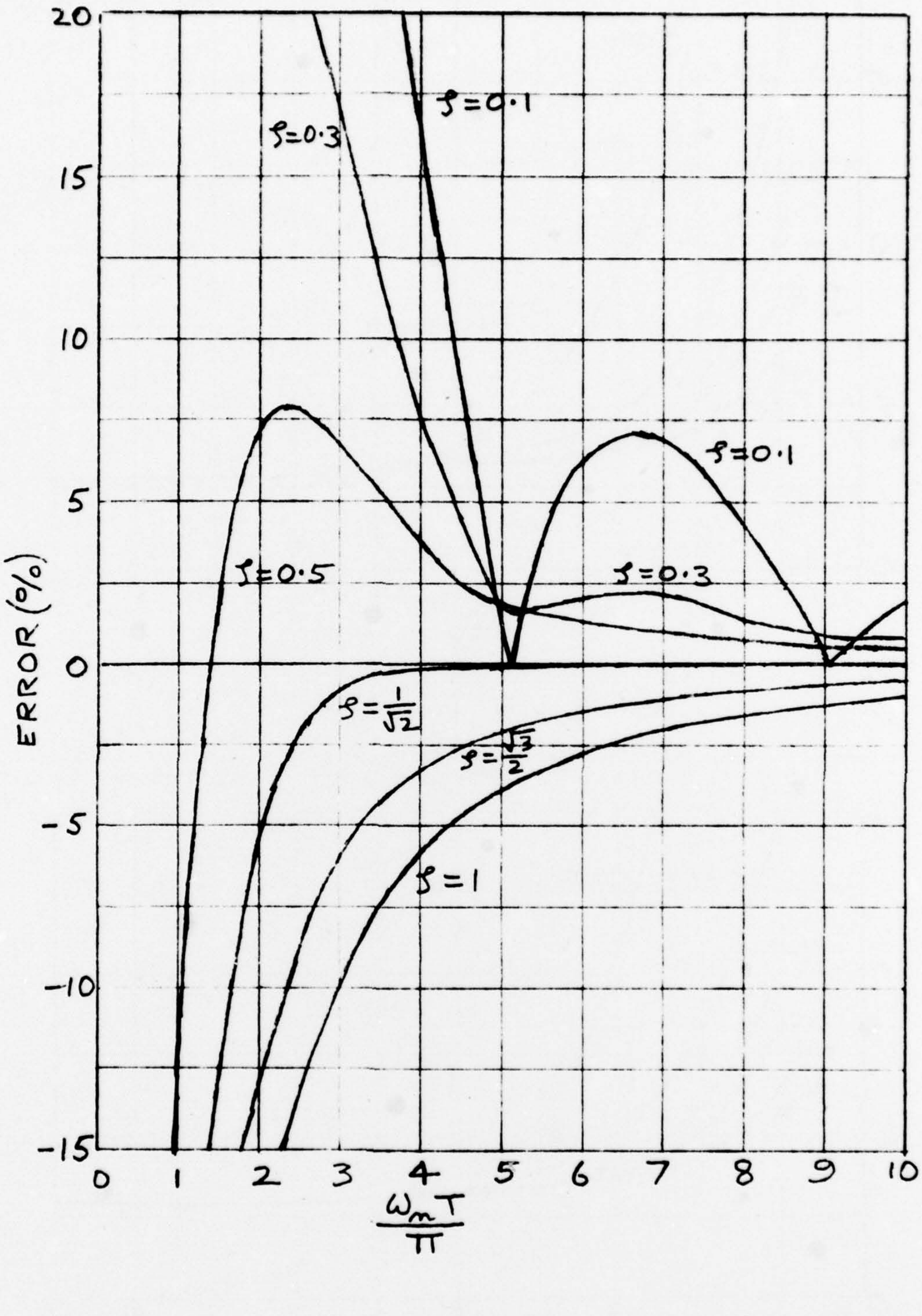
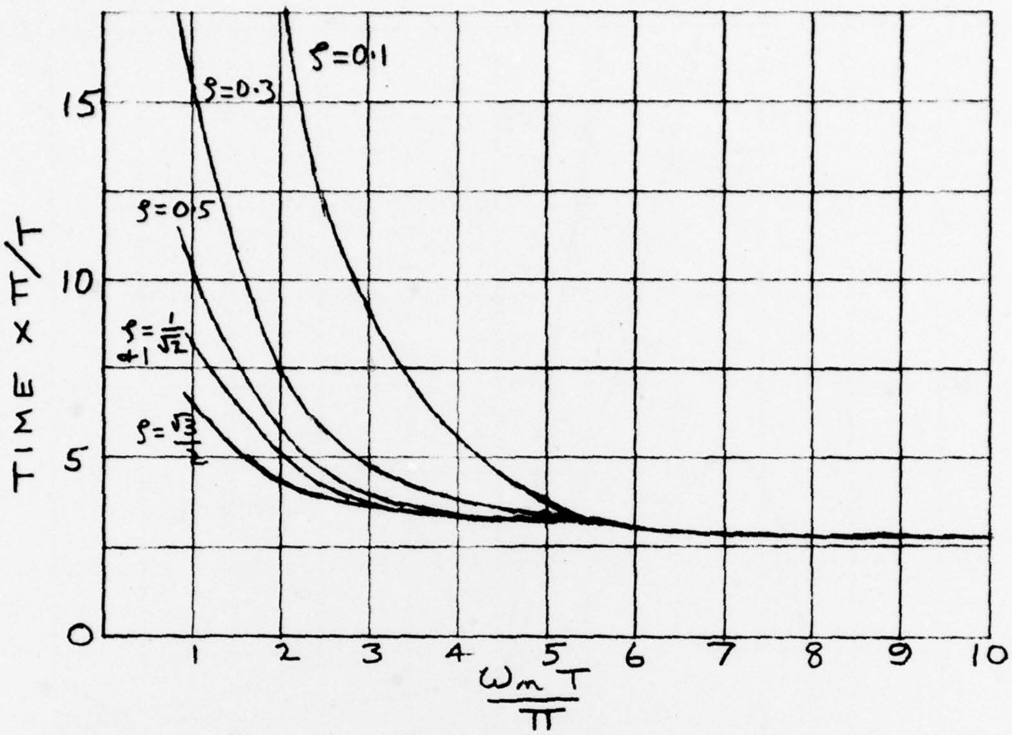
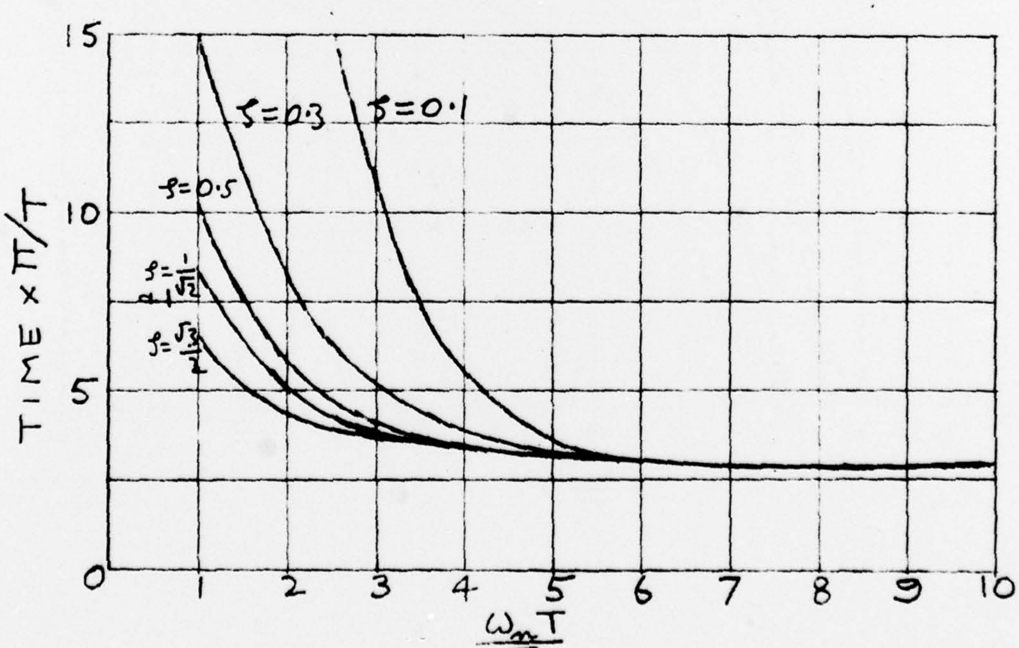


Fig 3 Error in measuring peak acceleration for ½ sin pulse

Fig 4a&b



a ½ SIN PULSE



b. SIN² PULSE

Fig 4 Time for integrated output to settle to within 1% of final value

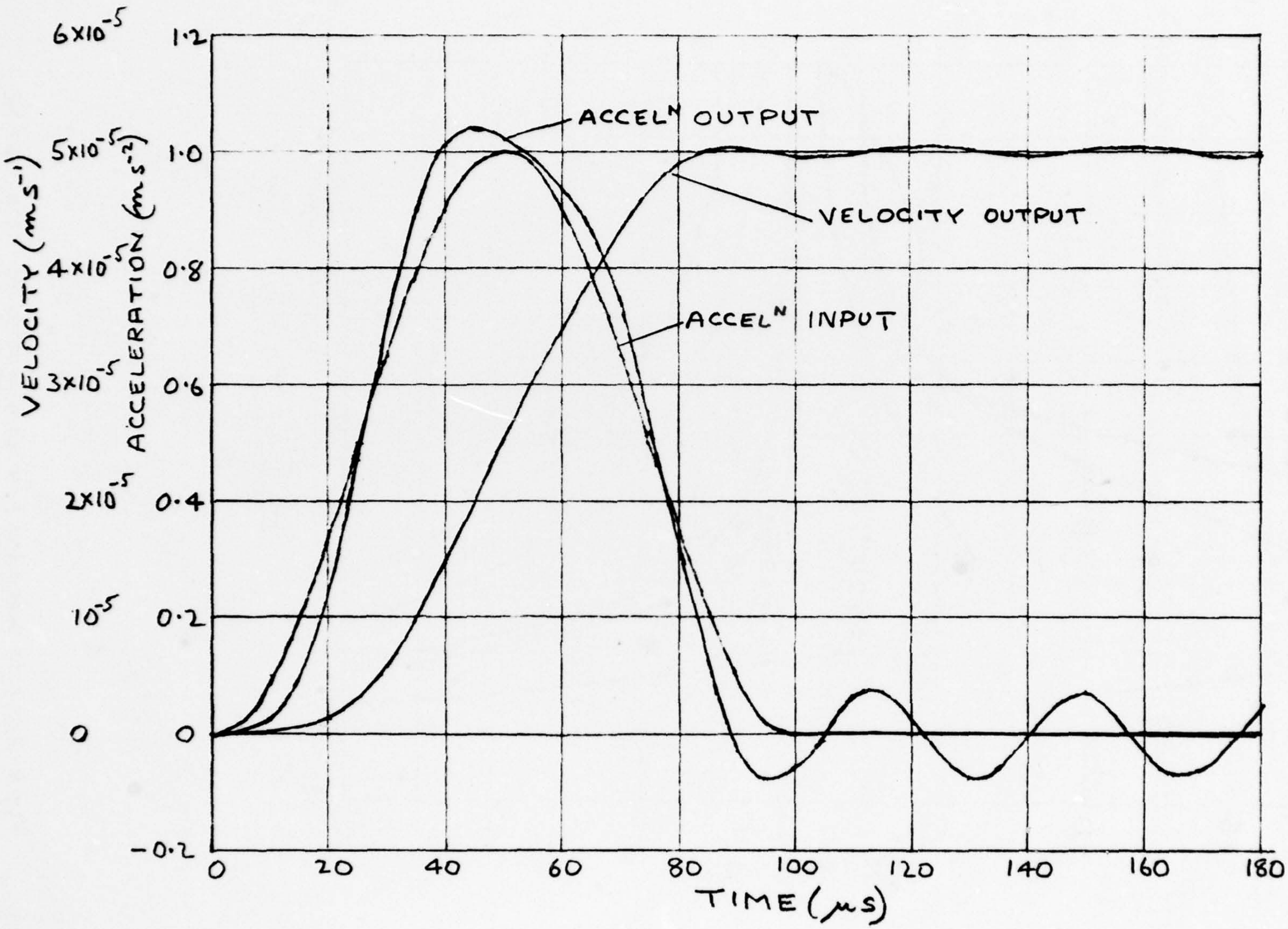


Fig 5 Response to $\sin^2$ pulse (T = 100 μs, f_n = 28 kHz and ζ = 0.01)

Fig 6

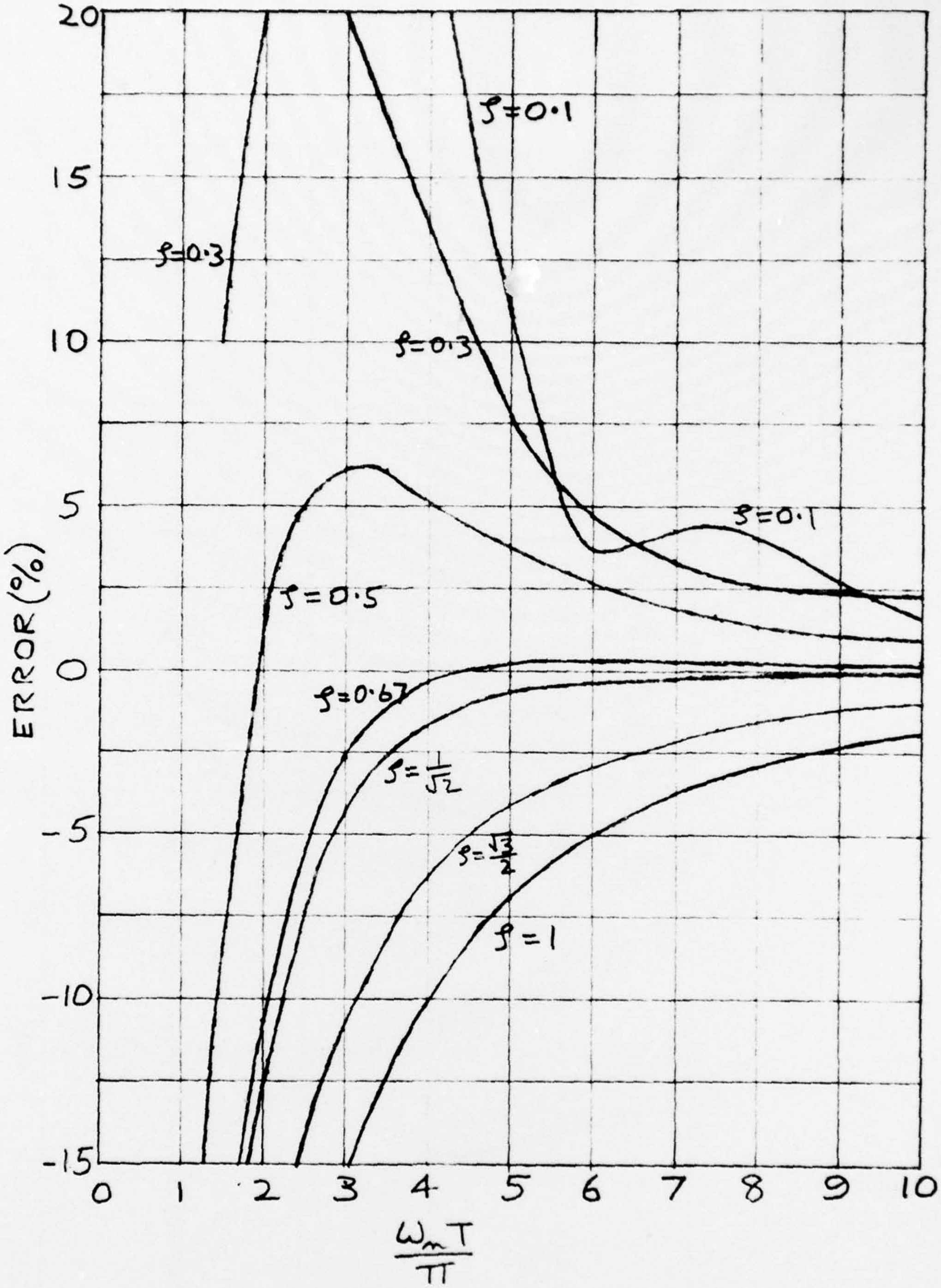


Fig 6 Error in measuring peak acceleration for $\sin^2$ pulse

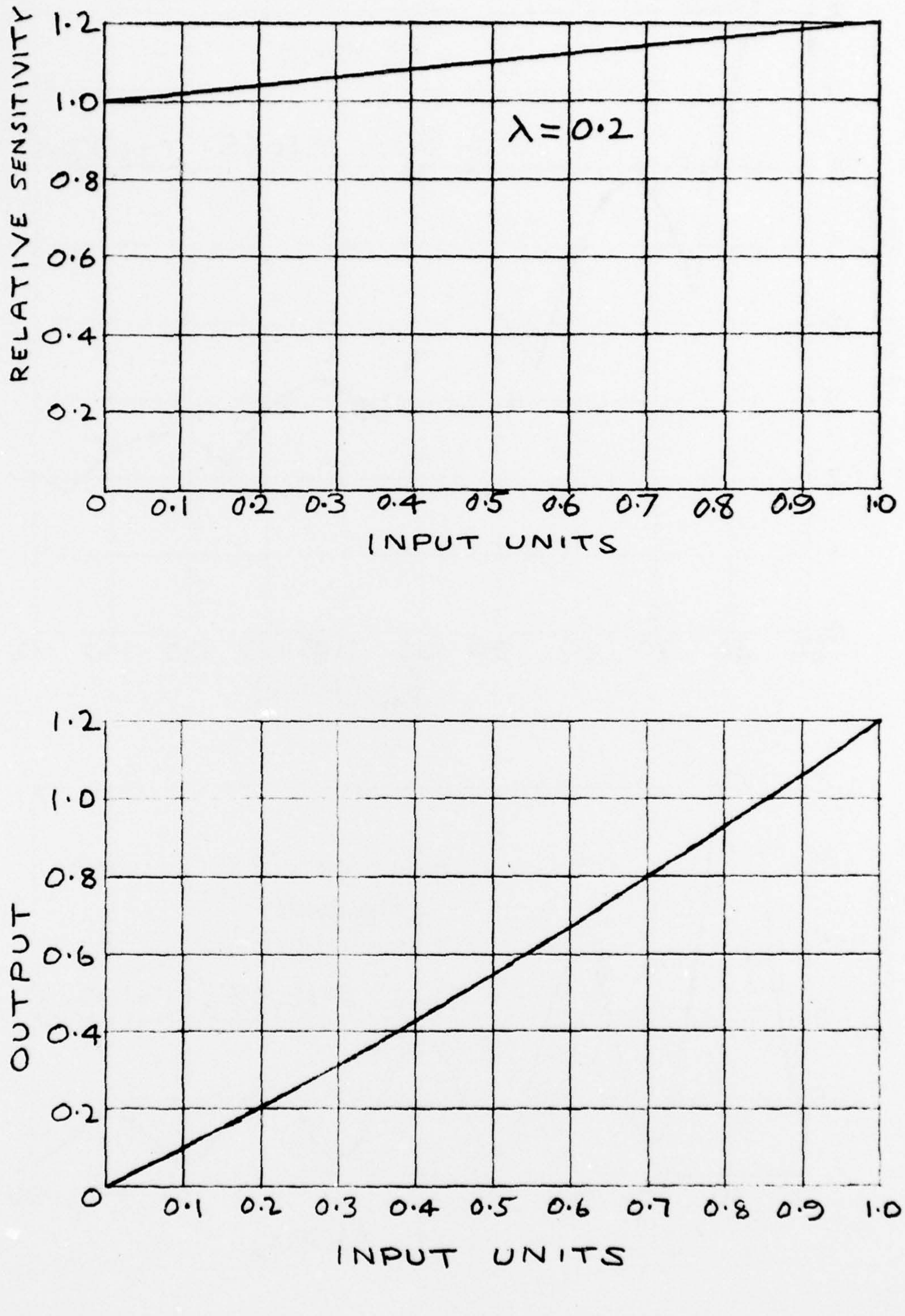


Fig 7 Nature of non-linearity

Fig 8

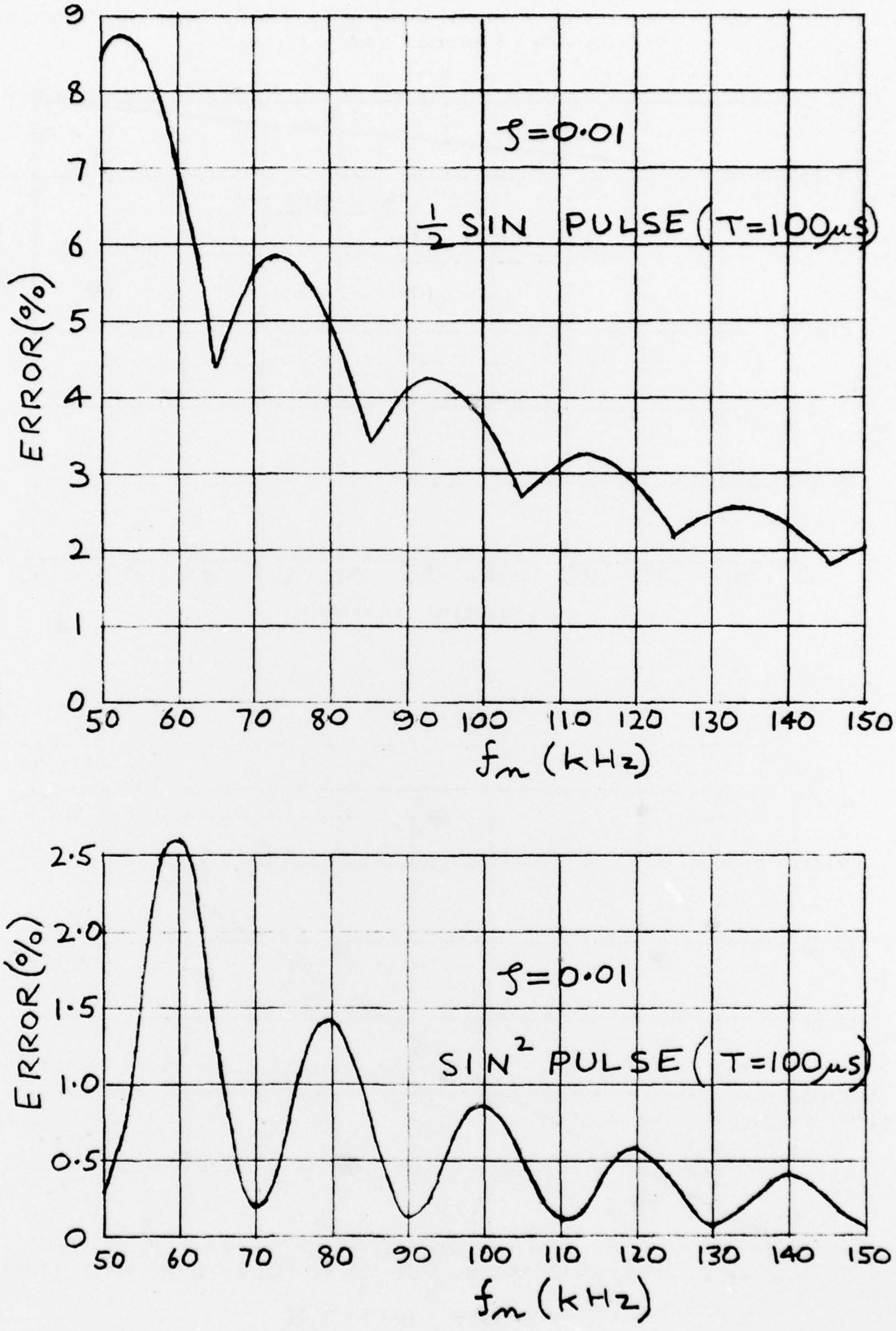


Fig 8 Error in measuring peak acceleration as a function of transducer natural frequency

REPORT DOCUMENTATION PAGE

Overall security classification of this page

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Macdonald, W.R.			January 1978	27	2

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17. Abstract

The response of accelerometers, with second-order filters, is considered for ½ sin and sin² acceleration pulses. The time history of their integrated (velocity) response is also treated.

Graphs relating the error in measuring peak acceleration to the filter parameters are given, and the results are related to practical situations experienced when calibrating accelerometers in RAE.

The effect of accelerometer non-linearity when calibrating by velocity-change methods is briefly investigated and a method of measuring the non-linearity is suggested.

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